EMERGENCE OF HIERARCHY IN NETWORKED ENDORSEMENT DYNAMICS

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SIAM Workshop on Network Science 2020
July 9–10 · Toronto

Summary

Recent work has documented the presence of prestige hierarchies in faculty hiring, with over 93% of sitting faculty in Computer Science, Business, and History, receiving a PhD from among the most prestigious quartile of institutions [2]. Yet, the fundamental mechanisms responsible for the emergence and reinforcement of prestige hierarchies are not well known. Here, we consider whether simple models of department and candidate decision-making are sufficient to reproduce quantifiable characteristics of real faculty hiring hierarchies. When compared to real hiring data, such a model would provide a more mechanistic understanding of the foundations of observed structures, shed light on the sources of variability in hierarchies across academic fields, and cautiously open the door to interventions designed to adjust the steepness of the current systematic inequalities of prestige.

In this vein, we study a simple model of prestige-reinforcement in which agents endorse other agents in response to their perceived position in an inferred hierarchy.

Network Endorsement Model

We define a simple stochastic process to model the emergence of hierarchy in networked endorsement dynamics. The system state at time t is encoded by a directed matrix $\mathbf{A}^{(t)}$ of endorsements. For example, $\mathbf{A}_{ij}^{(t)} = 2$ means that agent j endorsed agent i twice before time t. At each time-step, all n agents compute ranks $\mathbf{s}^{(t)} = \phi(\mathbf{A}^{(t)})$, where $\phi : \mathbb{R}^{n\times n} \to \mathbb{R}^n$ is a *ranking function* to be specified. The system updates according to

$$\mathbf{A}^{(t+1)} = \lambda \mathbf{A}^{(t)} + (1-\lambda)\boldsymbol{\Delta}(\mathbf{s}^{(t)}) \,, \tag{1}$$

where $\lambda \in [0,1]$ is a memory parameter governing the relative importance of new and old endorsements in the system state and $\boldsymbol{\Delta}$ is a random matrix function $\boldsymbol{\Delta} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ whose specification governs the dependence of the system update on the ranks $\mathbf{s}$. We can interpret λ as regulating the half-life of information in this system: after $\tau = \frac{\log(1/2)}{\log \lambda}$ rounds, the contribution of a given update to the current state is reduced by approximately half.

To instantiate this model, we choose the popular SpringRank functional [3] for ϕ, and let the choice function be given by $e^{\beta\phi_j}$. Linear stability analysis reveals the existence of two distinct regimes depending on the sign of $\beta - 2$. In the subcritical regime when $\beta < 2$ the egalitarian state in which all ranks are equal is stable. In this case, the system varies stochastically in the neighborhood of the egalitarian solution. In contrast, when $\beta > 2$, the egalitarian solution is unstable and persistent hierarchies emerge, with rare hierarchical inversions due to stochastic forcing. These regimes are separated by a critical phase at $\beta = 2$ in which hierarchies spontaneously emerge and dissolve over short timescales. Examples of each of these cases of β are shown in Figure 1.

Data Analysis

We use a standard maximum-likelihood method to estimate parameters β and λ. Initial studies of fitting our model to real world data are promising. A first study is on the directed network of PhD exchange in the US, data collected and analyzed by the authors of [4]. The dataset comes from The Mathematics Genealogy Project [1], which tracks Math PhD Graduates and their students. More precisely, an edge $i \to j$ translates to graduating PhD's at university j who later are a PhD advisor to a graduate at university i. We restrict our analysis to the 70 universities who are in a connected component during the time period 1960-2005.

We first estimated the model parameters β and λ, as shown in the top panel of Figure 2. The likelihood is peaked around our estimates of $\hat{\beta} = 2.32$ and $\hat{\lambda} = 0.87$. Since $\hat{\beta} > 2$, we are in the supercritical regime of stable hierarchy, although the low value of $\hat{\lambda}$ implies that the transition between regimes is not sharp. The inferred half-life of information in the network is $\frac{\log(1/2)}{\log \hat{\lambda}} \approx 5$ years. This characteristic timescale may reflect standard times to PhD completion and tenure promotion, although this connection remains speculative.

The relatively low value of $\hat{\lambda}$ implies that, while the presence

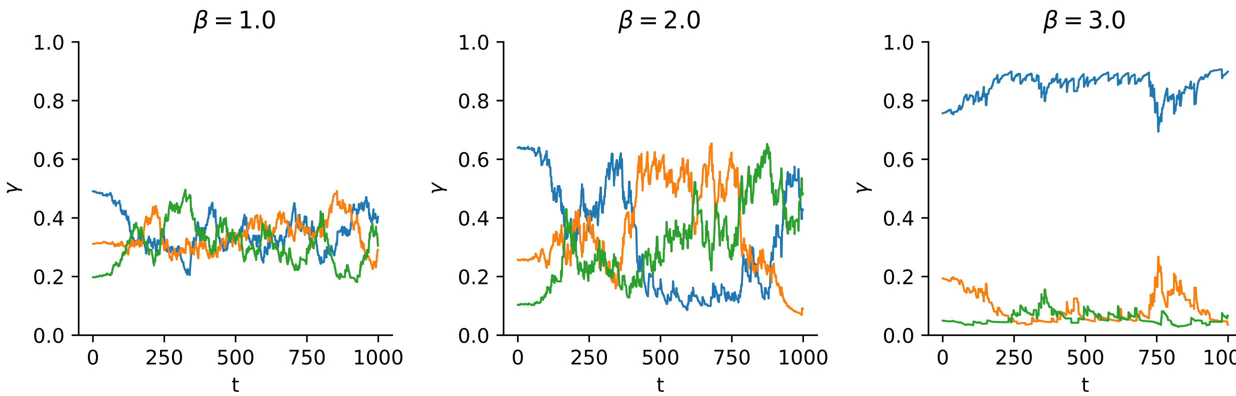

Figure 1: The rankings, γ, of 3 agents over time, t. In (a), when $\beta < 2.0$. the rankings are essentially equal and stable over time. In (c), when $\beta > 2.0$, one agent gains prestige over the others. In (b), when $\beta = 2.0$, different agents emerge with the highest rankings at different times.

of a hierarchy is stable, the rankings of specific schools are not. Figure 2 plots the trajectories in γ over the studied time-period. We have highlighted the six institutions with highest time-averaged entry of γ. In this time period, three distinct schools (Princeton, Harvard, and Stanford) occupy the top ranking. Averaged over time-steps, the institution at the top of the hierarchy produces PhDs at hiring institutions at a rate of over 60 times that of the lowest.

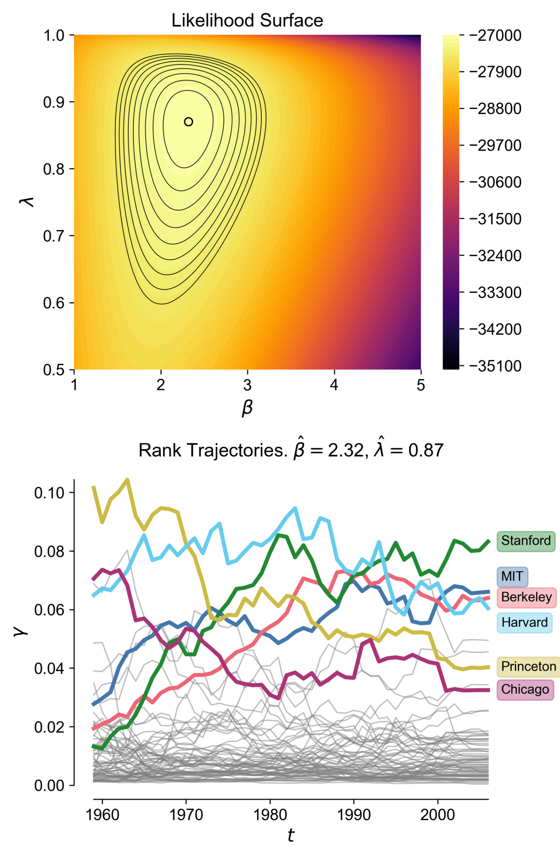

Figure 2: Analysis of the US PhD exchange in mathematics. (Top): Maximum likelihood inference of the parameters λ and β. (Bottom): Trajectories of γ in the time-period 1960-2005, using the maximum-likelihood parameters $\hat{\beta}$ and $\hat{\lambda}$.

Acknowledgments

We are grateful to the organizers of the Complex Networks Winter Workshop in Quebec City, in December 2019, at which this work was conceived.

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