ANALYZING SIGNED NETWORKS OF POLITICAL COLLABORATION BASED ON BALANCE AND CLUSTERABILITY

Samin Aref and Zachary Neal

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Summary

We use mathematical programming models to study signed networks of political collaboration and opposition in the US Congress similar to a parliamentary body, where several opposing coalitions may exist.

Keywords:

Political networks
Signed graphs
Mathematical modeling
Optimization
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Integer programming

Extended abstract

A *signed graph* is a graph with positive and negative signs on the edges usually denoted as $G = (V, E, \sigma)$ where V and E are the sets of vertices and edges respectively, and σ is the sign function $\sigma : E \to \{-1, +1\}$.

A signed graph (network) is *balanced* if its set of vertices, V, can be partitioned into two subsets $X \subseteq V$ and $\overline{X} = V \backslash X$ such that each negative edge joins vertices belonging to different subsets and each positive edge joins vertices belonging to the same subset [7]. If a signed network satisfies the same condition when partitioned into k subsets, it is called *clusterable* (k-balanced) [9].

These two definitions are more often expressed in terms of the classic definition of balance theory (attributed to the works of Heider in 40's [13] and that of Cartwright and Harary in 50's [7]) and the generalized definition of weak balance (by Davis in 60's [9]); which consider different types of cycles to be permissible. Balance theory [7] defines a balanced network as one where there is no cycle whose product of signs is negative. Generalized balance theory [9] defines a clusterable network as one with no cycle containing exactly one negative edge.

Signed networks representing real data often do not satisfy conditions of these theories [5]. This motivates analyzing them based on their distance to balance [4] and clusterability. Among different methods for measuring such distance is the minimum number of edges whose removal makes a network balanced (*frustration index* $L(G)$ [12, 16, 1]), k-balanced for the given value k (*k-clusterability index* $C_k(G)$ [11]) or clusterable (*clusterability index* $C(G)$ [8]).

Fig. 1(A) shows an example signed graph in which the five dotted lines represent negative edges and the two solid lines represent positive edges. The signed graph can be evaluated using 3-cycles (B), bi-partitioning (C), or k-partitioning (D). The first approach, (Fig. 1B), involves identifying triangle 1-4-5 as unbalanced and triangle 1-3-4 as balanced and only provides limited insight into the overall structure. The second approach, (Fig. 1C), involves finding a bi-partitioning of vertices $\{\{1, 2, 3\}, \{4, 5\}\}$ (shown by green and purple colors in Fig. 1C) which minimizes the total number of intra-group negative and inter-group positive edges to 1 ($L(G) = 1$). Note that removing edge $(4, 5)$ leads to a balanced signed graph. The last approach, (Fig. 1D), involves finding an optimal k-partitioning for the vertices $\{\{1, 2, 3\}, \{4\}, \{5\}\}$ which satisfies the conditions of generalized balance ($C(G) = 0, k^* = 3$).

Using the above definitions and concepts, we analyze signed networks of US Congress legislators [14, 15] in the Senate and House of Representatives based on their clusterability for different number of subsets. To be more precise, we use exact optimization models (mixed-integer and binary linear programming models) [10, 2] to compute clusterability indices of the networks and analyze optimal partitionings. Substantively, this analysis allows examining the US Congress similar to a parliamentary body, where coalitions form to achieve a majority voting bloc.

Previous studies on the same data show that signed networks of US Congress are close to being balanced according to cycle-based measures [15, 4] and optimally bi-partitionings of the networks [5, 3]. We use more general

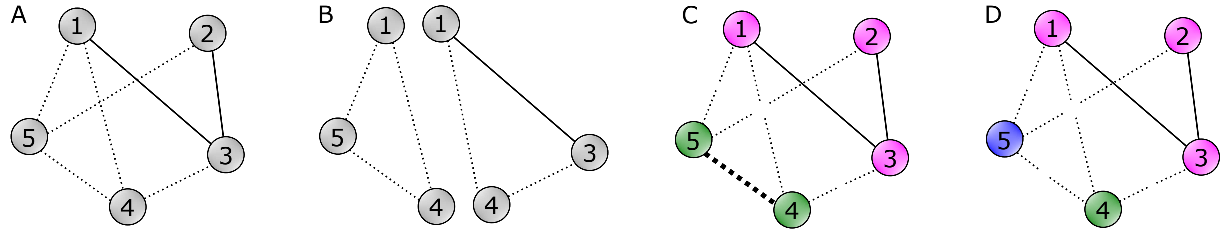

Figure 1: (A) An example signed network. (B) Evaluating balance using 3-cycles. (C) Evaluating balance via bi-partitioning. (D) Evaluating generalized balance and clusterability via k-partitioning

mathematical models for (1) specific pre-defined values of k [2] and (2) general value of k^* [6, 10] which compute the k-clusterability and clusterability indices respectively.

Our numerical results show that the clusterability indices of US Congress signed networks initially decrease for $k > 2$ which suggest that the signed ties between legislators in the US Congress are more consistent with a parliamentary-style set of coalitions than with a more conventional two-group categorization. We also obtain the globally minimum number of groups which minimize the clusterability index of each network by grouping legislators into a relatively large number of clusters $k^* >> 2$.

The results demonstrate that signed network of US Congress can be partitioned into coalitions exhibiting generalized balance and that the coalitions are not strictly related to party membership. The initial decline of clusterability index when k is gradually increased shows that the networks are more consistent with generalized balance than classic balance theory. However, clusterability index increases if the number of pre-defined groups is set to a value larger than k^* which seems to suggest that political collaborations prevents legislators from forming too many opposing sides.

Our observations show that legislators in US Congress seem to act more like a parliamentary body with more than two coalitions, but that these coalitions still mirror broad liberal-conservative tendencies. In this presentation, we provide our results on clusterability of US Congress network and the dynamics of its major coalitions over 1979-2018.

References

[1] S. Aref, A. J. Mason, and M. C. Wilson. Computing the line index of balance using integer programming optimisation. In B. Goldengorin, editor, *Optimization Problems in Graph Theory*, pages 65–84. Springer, 2018.

[2] S. Aref, A. J. Mason, and M. C. Wilson. A modeling and computational study of the frustration index in signed networks. *Networks*, 75(1):95–110, 2020.

[3] S. Aref and Z. Neal. Detecting coalitions by optimally partitioning signed networks of political collaboration. *Scientific Reports*, 10(1):1–10, 2020.

[4] S. Aref and M. C. Wilson. Measuring partial balance in signed networks. *Journal of Complex Networks*, 6(4):566–595, 2018.

[5] S. Aref and M. C. Wilson. Balance and frustration in signed networks. *Journal of Complex Networks*, 7(2):163–189, 08 2019.

[6] N. Bansal, A. Blum, and S. Chawla. Correlation clustering. *Machine learning*, 56(1-3):89–113, 2004.

[7] D. Cartwright and F. Harary. Structural balance: a generalization of Heider's theory. *Psychological Review*, 63(5):277–293, 1956.

[8] D. Cartwright and F. Harary. Balance and clusterability: An overview. In P. W. Holland and S. Leinhardt, editors, *Social networks: Surveys, advances, and commentaries*, pages 25–50. Academic Press, 1976.

[9] J. A. Davis. Clustering and structural balance in graphs. *Human relations*, 20(2):181–187, 1967.

[10] E. D. Demaine, D. Emanuel, A. Fiat, and N. Immorlica. Correlation clustering in general weighted graphs. *Theoretical Computer Science*, 361(2):172–187, 2006.

[11] P. Doreian and A. Mrvar. A partitioning approach to structural balance. *Social networks*, 18(2):149–168, 1996.

[12] F. Harary. On the measurement of structural balance. *Behavioral Science*, 4(4):316–323, 1959.

[13] F. Heider. Social perception and phenomenal causality. *Psychological Review*, 51(6):358–378, 1944.

[14] Z. Neal. The backbone of bipartite projections: Inferring relationships from co-authorship, co-sponsorship, co-attendance and other co-behaviors. *Social Networks*, 39:84–97, 2014.

[15] Z. Neal. A sign of the times? Weak and strong polarization in the U.S. Congress, 1973–2016. *Social Networks*, 60:103 – 112, 2020.

[16] T. Zaslavsky. Balanced decompositions of a signed graph. *Journal of Combinatorial Theory, Series B*, 43(1):1–13, 1987.